A short note on stereovision with an aligned camera pair : a guide for selecting the baseline and the cameras field of view

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Abstract

This note deals about some geometric facts on stereovision when the two cameras are perfectly aligned. It gives some clues on how to select the cameras field of view and baseline, considering a given application context. It covers the topic of maximal error on measured distance, and the image overlapping problem. It does not consider the matching step.

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1 Introduction

In the field of Computer Vision, stereovision can be achieved with two cameras in almost any positions, but the so-called *aligned position* can greatly simplify further processing. This position is defined by parallel optical axis (ususually noted as the Oz axis), and by coincident horizontal axis of the two cameras (see fig. 1).

The two main advantages are as follow. First, it simplifies the matching step, as stereo-correspondant points lie on the same line. Second, 3D reconstruction is immediate, as it can be demonstrated that the distance of a 3D point is directly related to the difference of the horizontal positions of the two projections of this point (aka *disparity*).

The user needs to set the baseline (distance between the cameras axis), and to select a field of view for the cameras. The paper will briefly recall some basic facts about aligned stereo geometry, and will show some equations and graphs that will help a user to understand how he must select these parameters. It will consider the case of a typical industrial application, where some scene is viewed with a constant "reference" distance z_0 (a plane perpendicular to the optical axis), with the need of detecting and/or measuring objects that will lie between this reference plane and the camera pair.

In some situations, one or more of the parameters are fixed, for example if you use an integrated rig such as those shown on fig. 2. This paper can help you for evaluating the practical characteristics (overlapping and error) of your system.

2 Aligned stereovision geometry

When the two cameras are aligned such as described by fig. 1, computing distance once we have established correspondence is simple. As it can be seen on fig. 3, the relation between distance z and disparity d is given by (1), with f representing the focal, and u and u' the horizontal positions of the two points m and m'. These are the projections of the same 3d point M in the two images.

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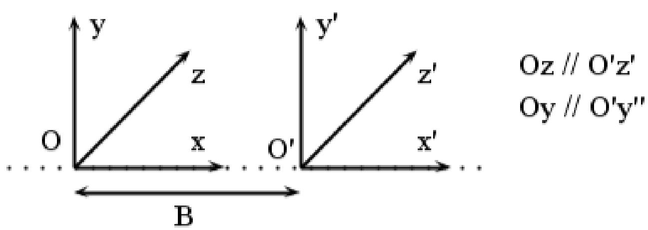


Figure 1: Aligned cameras configuration. The Oz axis is the optical axis of the camera.

(a) Elphel camera set (`www3.elphel.com`).

(b) Bumblebee 2 camera set (`www.ptgrey.com`).

Figure 2: An example of two commercial products available, that both have a **fixed** baseline.

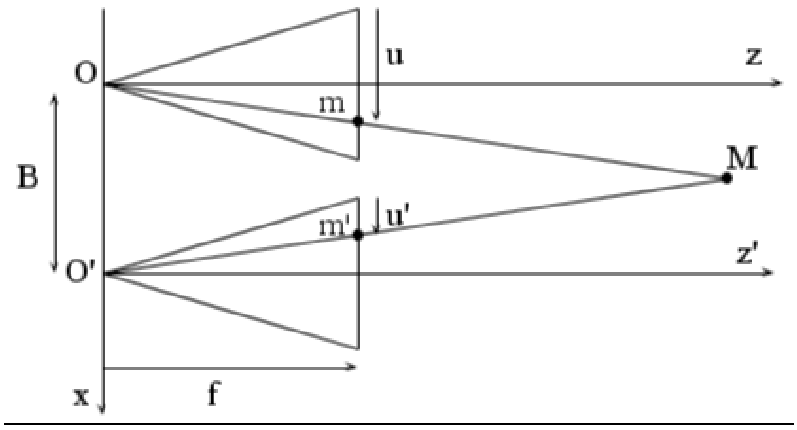


Figure 3: Projection of a 3D point M into the two projections planes in the aligned camera rig.

$$z_0 = b.f/d \quad \text{with} \quad d = u - u' \tag{1}$$

Recall that we are here in metric units (meters). It is of course more convenient to express d with pixels rather than meters, so we need to include a scale factor, usually noted k (pixels/m.). Sometimes, camera vendors provide instead the size of the sensors pixels (p), usually around 4 to 8 μm. These values get included into a global scale factor $\alpha = k\,f$ or $\alpha = f/p$, that can be used to express the distance of a point using (2).

$$z_0 = \frac{b.\alpha}{d} \tag{2}$$

Scale factor For example, the Sony camera ref. XCD-MV6[1] has a pixel size of 6μm. If this camera is mounted with an objective that has a focal length of 8mm, then we will have $\alpha = 1333$.

3 Error on the measured distance

Obviously, the farther the point is from the cameras, the more sensitive will the measure be: a small variation of disparity will produce a large variation on computed distance. If matching is done on the pixels (as opposed to a sub-pixel matching strategy), then there will allways be an error e on the disparity, that will be between 0 and 0.5 pixels. So we can model the measured distance of a random 3D point by adding the pixel error e to disparity:

$$z = \frac{b.\alpha}{d + e}$$

The relative error on the measured distance for a given reference distance z_0 having a null error will then be expressed by (3). [2]

$$\epsilon(z_0) = \frac{|z - z_0|}{z_0} = \frac{e}{e + d} \tag{3}$$

[1] See `www.pro.sony.eu/vision`

[2] We can drop the "absolute" operator, as error e and disparity d are defined to be positive.

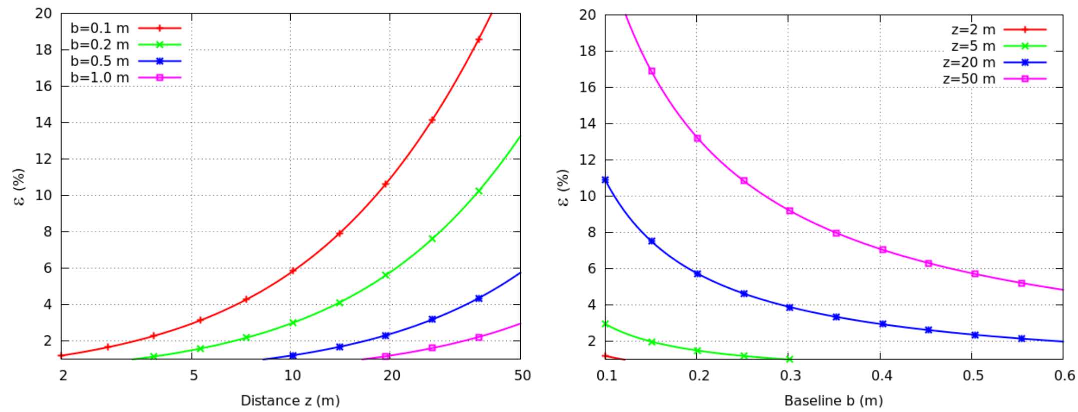
Replacing disparity d with $b\,\alpha/z_0$, we can express $\epsilon(z_0)$ using b, α and z_0:

$$\epsilon(z_0) = \frac{e.z0}{b.\alpha + e.z_0}$$

Finally, if we consider the maximum pixel error $e = 0.5$, the error is given by (4).

$$\epsilon(z_0) = \frac{1}{1 + \dfrac{2\,b\,\alpha}{z_0}} \tag{4}$$

We show on figure fig. 4 the maximum value for this error for some typical camera parameters taken from the Bumblebee2 product (pixel size of 4.65 μm, and focal of 3.8mm). As expected, a larger baseline reduces this error, for a given distance z_0. However, recall that this does **not** take into account potential matching errors.



(a) Error related to distance of considered scene element, for several baseline values.

(b) Error related to baseline, for several distance values.

Figure 4: Maximal systematic error for a integer-type matching strategy.

Conclusion These plots clearly show that a "pixel" matching approach can lead to high error rates on the computed distance, particularly when distance to the camera increases. This can be explained by the fact that for high distances, true disparity gets close to 0, so the potential half-pixel error produces a high error on the computed distance. Therefore, subpixel matching strategies are highly recommended. However, keep in mind that even with sub-pixel matching, error on the computed distance will get higher as distance increases.

4 Overlapping of images

As demonstrated in the previous section, for a given distance, the error will be lower when the baseline increases. However, this has a practical limit: overlapping of the images has to be sufficient so matching can occur. For a given horizontal field of view (FOV[3]) β, if the baseline gets too high, then there will be no common region to produce the matches. This is illustrated by fig. 5(a): it is clear that if the baseline b increases two much, the common region will disappear. Usually, we need at least 60 to 80% of overlapping, but this is depending on the application.

In order to give a model of this overlapping, we consider the scene as a plane at a distance z_0 of the camera set. This will of course not correspond to most applications, but can still be useful: one just has to consider the usual distance of the main object to be considered in the scene.

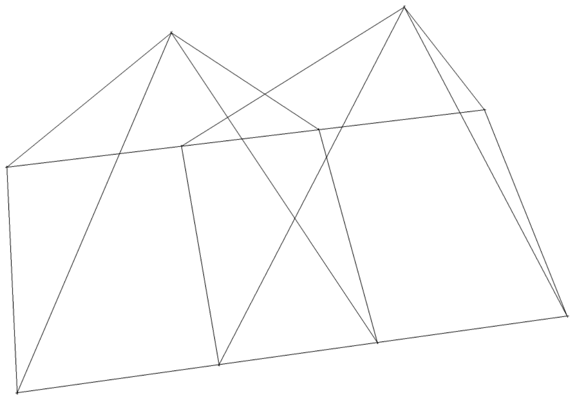
We define the overlapping ratio r as the ratio between the common part of the z_0 plane and the whole size of the scene (image width). Of course, this makes senses only if there **is** some overlapping. Using fig. 5(b), this can be expressed as a condition on z_0: it must be high enough so that images *do* overlap. Formally, this can be written as condition (5), with β the horizontal FOV of the camera.

$$z_0 > z_{\min} \qquad \text{with} \qquad z_{\min} = \frac{b}{2.\tan(\beta/2)} \tag{5}$$

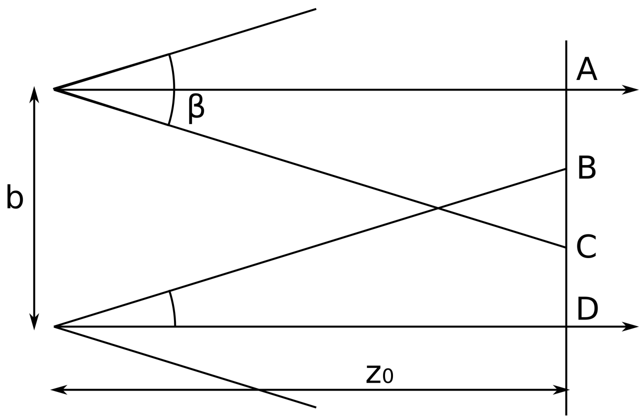
We can express the overlapping ratio by (6):

$$r = \begin{cases} \dfrac{BC}{2\;AC} & \text{if} \quad z_0 > z_{\min} \\ 0 & \text{if} \quad z_0 \leq z_{\min} \end{cases} \tag{6}$$

[3]See en.wikipedia.org/wiki/Field_of_view



(a) 3D view of the two images.



(b) Top view of the two images.

Figure 5: Overlapping of the two images.

	image width (pix.)		
focal length	400	640	720
4 mm	35	60	69
6 mm	23	38	43
8 mm	17	28	32

Table 1: Example of FOV angles in degrees for some common camera values (with pixel size $p = 6$mm).

We can also see that $AB = b - BD = b - AC$ (as we have of course $BD = AC$). So we have $BC = AC - AB = AC - b + AC = 2.AC - b$. Assuming that the condition (5) is true, the ratio r can be written as:

$$r = \frac{BC}{2\,AC} = \frac{2\,AC - b}{2\,AC} = 1 - \frac{b}{2\,AC}$$

Basic trigonometric knowledge gives us: $AC = z_0\ \tan(\beta/2)$. So finally:

$$r = 1 - \frac{b}{2\,z_0\ \tan(\beta/2)} \tag{7}$$

Usually, the angle β is given by the camera vendor, but it can be easily deduced from pixel size p, image width w, and the focal length f, using formula (8). For example, the table 1 gives some values of β for different values of focal lengths and image sizes, and a pixel size $p = 6$mm.

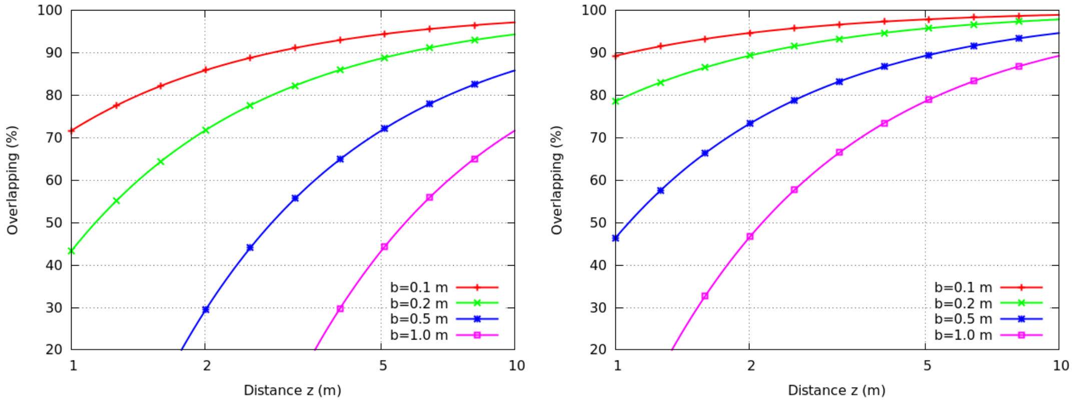
$$\beta = 2 \tan \frac{w\,p}{2\,f} \tag{8}$$

We show on fig. 6 the overlapping ratio related to the reference distance z_0, for several values of the baseline and two values of field of view β. We show on fig. 7 this ratio related to baseline for several field of view values. As expected, it decreases linearly.

We produce on figures 8, 9,10 plots of constant overlapping ratio curves. These can be used to select the parameters of the camera system (baseline, camera field of view), knowing the application reference distance z_0 and the desired overlapping ratio. For other scale factor values, they can be easily be reproduced using any good plotting program[4].

Usage These figures can be used in the following way. For example, say your image resolution and focal length is fixed, in a way that you have an horizontal FOV equal to 60°, and you want to know what baseline you need, in order to get a given overlapping at a given distance. Let's say 70% at 2m. Fig. 10(b) shows that in that case, you need a baseline of 0.7m, and that if objects in the scene are at 3m instead, then you will reach 80% of overlapping.

[4]These were done using Gnuplot (`www.gnuplot.info`).



(a) $\beta = 20°$ (b) $\beta = 50°$

Figure 6: Overlapping ratio of the two images related to distance z, for several baseline values.

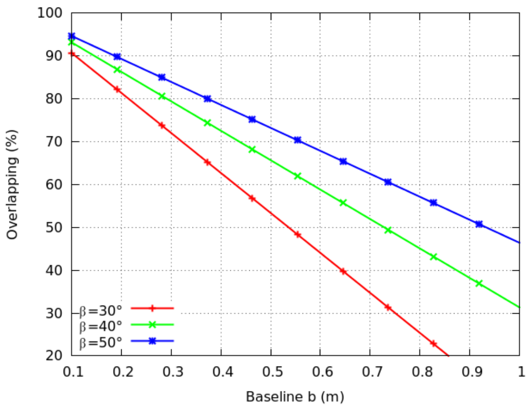
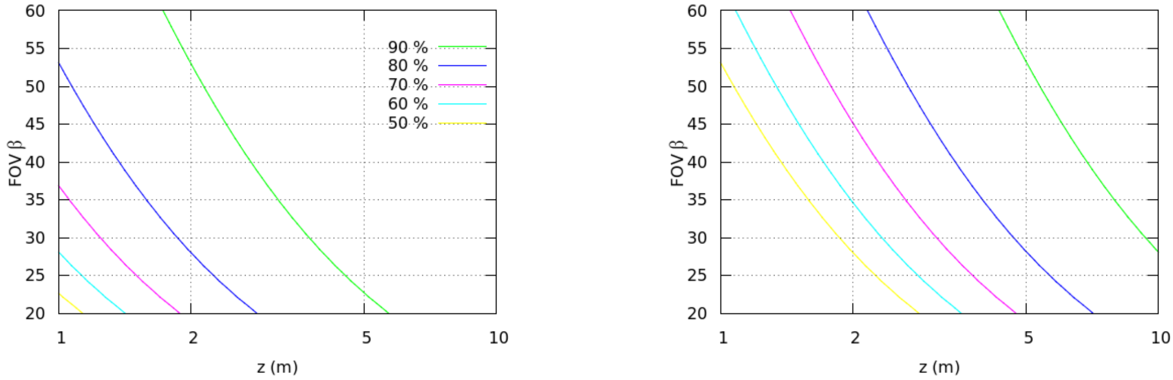
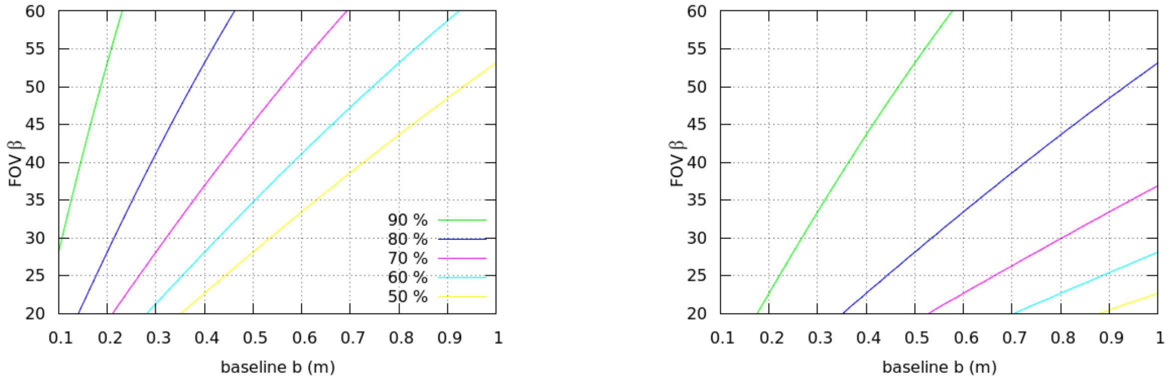


Figure 7: Overlapping ratio of the two images related to baseline, for several values of the field of view of the camera, and for a distance $z = 2$m.



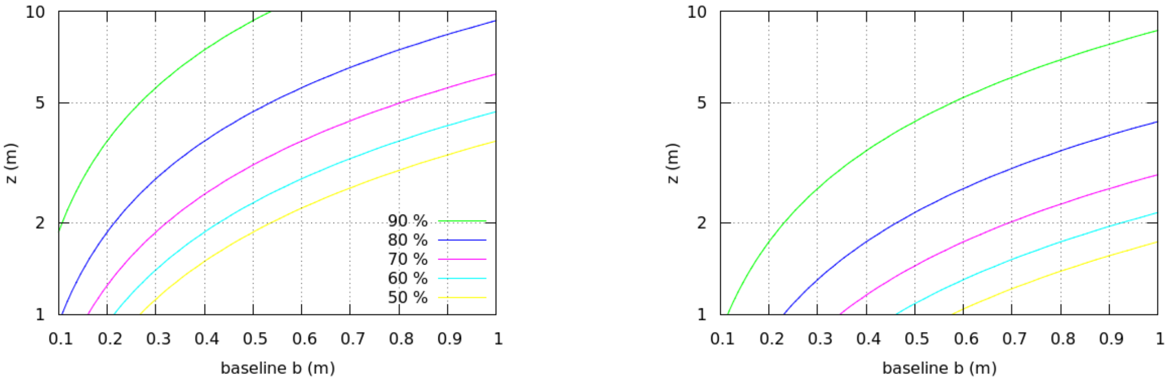
(a) Baseline $b = 0.2m$ (b) Baseline $b = 0.5m$

Figure 8: Constant overlapping ratio curves related to camera field of view β and distance z, for two values of the baseline.



(a) Distance $z = 2m$ (b) Distance $z = 5m$

Figure 9: Constant overlapping ratio curves related to camera field of view β and baseline b, for two values of distance z.



(a) FOV β =30° (b) FOV β =60°

Figure 10: Constant overlapping ratio curves related to baseline b and distance z, for two values of FOV β.